

Phase diagram of chiral quark matter: color and electrically neutral Fulde-Ferrell phase

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The phase diagram of charge and color neutral two-flavor color superconducting quark matter is studied including the homogeneous two-flavor superconductor (2SC) and the inhomogeneous Fulde-Ferrell (FF) phases within the Nambu-Jona-Lasinio model. The low-temperature domain $T \leq 5$ MeV of the phase diagram contains the FF phase, which borders at high temperatures to the 2SC phase. The critical temperature of phase transition from the 2SC to the unpaired state is in the range 20-30 MeV. We derive the equation of state of matter and its composition and show that matter in mature compact stars should be in the inhomogeneous FF-like superconducting state. We briefly discuss the astrophysical implications of such a phase in compact stars.

I. INTRODUCTION

The quark substructure of baryons, predicted by quantum chromodynamics (QCD), suggests that nucleonic matter may deconfine at sufficiently high densities attainable in compact stars. Compact stars featuring quark cores, known as hybrid stars, may be distinguishable from their purely nucleonic/hyperonic counterparts by their astrophysical manifestations, which may include magnetic, thermal, and rotational evolutions on secular timescales, as well as shorter transients.

In this paper we derive the equation of state and composition of two-flavor dense quark matter at nonzero temperature under the conditions of charge neutrality and β equilibrium. At moderate densities quark matter is composed of up and down quark flavors only, which in newborn protoneutron stars occur at near equal amount (the isospin chemical is vanishingly small). However, rapid compression of matter leads to its neutronization via the inverse beta decay and the isospin chemical potential grows until it reaches the asymptotic value determined by the β -equilibrium condition $\mu_d - \mu_u = \mu_e$ among d and u quarks and electrons (here μ_i with $i = d, u, e$ are their chemical potentials, respectively). Therefore, under stellar conditions the pairing between the two light flavors of quarks occurs at finite isospin chemical potential, *i.e.*, when the Fermi surfaces of up and down quarks are shifted apart by an amount which could be of the same order of magnitude as the gap in the quasiparticle spectrum. Under these conditions the actual pairing pattern may be significantly different from that of the BCS and is likely to involve breaking of the spatial symmetries by the condensate order parameter, in analogy to less exotic low-temperature systems found in condensed matter (*e.g.*, electron gas or ultracold atomic vapor) [1–3].

The phase diagram of charge and color neutral quark matter, which features *homogeneous* condensates at non-zero temperature and moderate densities, is well established at the mean-field level [4–6]. It contains a sequence of phases with the two-flavor superconducting (2SC) phase at low densities and the color-flavor-locked (CFL) phase at asymptotically high densities (the color superconducting phases are reviewed in Ref. [7]; for a review of astrophysical implications of color superconductivity see, *e.g.*, Ref. [8, 9]). The domain occupied by the *inhomogeneous* phases is difficult to access in general, since most of the studies are carried out in the regime where the Ginzburg-Landau expansion of the mean-field thermodynamic potential is valid [10–14]. An exception is the so-called Fulde-Ferrell (or plane-wave crystalline color superconducting) phase [15–21], which admits straightforward treatment in the full domain of interest to compact stars. Low-dimensional treatments of inhomogeneous phases admit some simplifications as well [22]. In effective four-fermion coupling models, like the Nambu-Jona-Lasinio (NJL) model, the interactions can be easily varied in a broad range of (effective) couplings that may account for medium modifications of the interactions in dense quark matter.

In this paper we examine the phase diagram of charge and color neutral quark matter at non-zero temperatures and moderate densities and show that the simplest realization of the phases with broken spatial symmetries – the Fulde-Ferrell phase – arises in the low-temperature domain of the phase diagram. We do so, by extending the discussion in Ref. [20] to include the conditions of charge and color neutrality, which impose additional constraints on the set of equations that are solved. As indicated above, we have chosen the spatial dependence of the order parameter that corresponds to a single plane wave. Therefore, our calculations offer only an upper bound on the ground state energy, since more complicated order parameters have been proven to possess lower energy in the domain where the Ginzburg-Landau expansion is valid. However, qualitatively the phases with broken spatial symmetries share many common features and the Fulde-Ferrell (FF) phase is a convenient starting point of exploration.

Our discussion is restricted to the two-flavor case appropriate at moderate densities, *i.e.*, its validity domain is

constrained by the temperature and density where strange quarks appear in substantial amounts. The role of strange quarks is twofold: first, because of their negative charge it becomes energetically favorable to replace high momentum electrons by low momentum strange quarks; second, strange quarks disfavor the BCS type cross-flavor pairing among the light quarks and favors instead the FF or other crystalline-type pairing. If the Fermi energy of strange quarks is large enough they may choose to pair in a color-flavor-locked pattern which, except for the asymptotically high densities, will involve some mismatch between the Fermi surfaces with possibly multiple FF phases involving quarks of all three flavors.

Finally, a number of non-crystalline phases have been suggested in the literature, which address the stress caused by the mismatch of Fermi surfaces differently. These alternative phases invoke separation into normal and superconducting domains [23] or pairing induced changes in the shapes of the Fermi surfaces [24].

This paper is organized as follows. Section II presents the key equations describing the FF phase (see also Ref. [20]). In Sec. III we discuss the conditions of charge and color neutrality. Our results are presented in Sec. IV and our conclusions are collected in Sec. V.

II. GAP EQUATIONS AND THERMODYNAMICS OF FF PHASE

The mean field (MF) thermodynamic potential of the FF phase is given by

$$\Omega_{\text{MF}} = \Omega_{\text{MF}}^{\Delta} + \Omega_{\text{MF}}^0, \quad (1)$$

where the first term includes the contribution from the condensate of red-green quarks and the second term is the contribution from the unpaired blue quarks. The first term is given by

$$\Omega_{\text{MF}}^{\Delta} = 3 \frac{\Lambda^2 \Delta^2}{g^2} - 2 \sum_e \int \frac{d^3 k}{(2\pi)^3} \left\{ E_e^+(\Delta) + E_e^-(\Delta) - 2T \ln f[-E_e^+(\Delta)] - 2T \ln f[-E_e^-(\Delta)] \right\}, \quad (2)$$

where Λ is the cutoff of the NJL model, g is the strong coupling constant, $f(\omega) = [1 + \exp(\omega/T)]^{-1}$ is the Fermi distribution, and T is the temperature. The contribution of the blue quarks is

$$\Omega_{\text{MF}}^0 = -2 \int \frac{d^3 k}{(2\pi)^3} \sum_f \left\{ |\mathbf{k}| - T \ln f[-\beta(|\mathbf{k}| - \mu_b^f)] - T \ln f[-\beta(|\mathbf{k}| + \mu_b^f)] \right\}, \quad (3)$$

where f summation is over flavors and μ_b^f is the chemical potential of blue quarks of flavor f . The thermodynamic potential depends on the quasiparticle energies, which are given by

$$E_e^{\pm}(\Delta) \approx E_{A,e} \pm \sqrt{E_{S,e}^2 + |\Delta|^2}, \quad (4)$$

where $E_{S,e}$ and $E_{A,e}$ are the parts of the spectrum which are even (symmetric) and odd (asymmetric) under exchange of quark flavors in a Cooper pair. These are given by¹

$$E_{S,e}(|\mathbf{k}|, |\mathbf{Q}|, \theta, \bar{\mu}) = E^+ - e\bar{\mu}, \quad (5)$$

$$E_{A,e}(|\mathbf{k}|, |\mathbf{Q}|, \theta, \delta\mu) = \delta\mu + eE^-, \quad (6)$$

with $E^{\pm} = (|\mathbf{k} + \mathbf{Q}/2| \pm |\mathbf{k} - \mathbf{Q}/2|)/2$, where θ is the angle formed by vectors \mathbf{k} and \mathbf{Q} , $\delta\mu = (\mu_u - \mu_d)/2$ and $\bar{\mu} = (\mu_u + \mu_d)/2$. The spectrum of unpaired blue quarks is given by $\xi_e^b(\mathbf{k}, \mu_b) = E_{S,e}(\mathbf{k}, 0, 0, \mu_b) = |\mathbf{k}| - e\mu_b$, where μ_b is the chemical potential of blue quarks.

The form of quasiparticle spectrum requires a subtraction from the thermodynamic potential of the unphysical contribution from currents that exist in the case where pairing gap vanishes. Therefore, we need to evaluate

$$\Omega_{\text{MF}} = \Omega_{\text{MF}} - \Omega_{\text{MF}}^{\Delta=0} + \Omega_{\text{MF}}^{\Delta=0, \mathbf{Q}=0}. \quad (7)$$

¹ Because $|\mathbf{Q}| \sim \delta\mu$ is small compared with the typical scale $\bar{\mu}$ for quark momenta, we can keep the leading order in $|\mathbf{Q}|/\bar{\mu}$ terms, in which case the spectrum takes the form given in Ref. [20].

The stationary point(s) of the thermodynamic potential determine the equilibrium values of the order parameters:

$$\frac{\partial \Omega_{\text{MF}}}{\partial \Delta} = 0, \quad \frac{\partial \Omega_{\text{MF}}}{\partial |\mathbf{Q}|} = 0. \quad (8)$$

The direction of the vector \mathbf{Q} , *i.e.*, the axis of symmetry breaking is chosen by the system spontaneously; the appearance of a preferred direction implies that $O(3)$ symmetry of the BCS theory is broken down to $O(2)$ in the FF phases; phases with more complicated lattice-type order parameters break the translational symmetry of the problem. The relative concentrations of the u and d quarks are determined from the derivative of the thermodynamic potential with respect to the corresponding chemical potential as

$$\frac{\partial \Omega_{\text{MF}}}{\partial \mu_u} = n_u, \quad \frac{\partial \Omega_{\text{MF}}}{\partial \mu_d} = n_d. \quad (9)$$

The explicit form of the gap equation, which follows from Eqs. (2) and (8), is

$$\Delta = \frac{g^2}{3\Lambda^2} \sum_e \int \frac{d^3 p}{(2\pi)^3} \frac{\Delta}{E_e^+(\Delta) - E_e^-(\Delta)} \left\{ \tanh \left[\frac{\beta}{2} E_e^+(\Delta) \right] - \tanh \left[\frac{\beta}{2} E_e^-(\Delta) \right] \right\}. \quad (10)$$

Note that in the isospin-symmetric limit the quasiparticle spectra are degenerate $E_e^+(\Delta) = -E_e^-(\Delta)$ and (with $|\mathbf{Q}| = 0$) the gap equation reduces to the ordinary BCS gap equation for ultrarelativistic fermions. Finally, the equation for the total momentum is given by

$$0 = - \sum_e \int \frac{d^3 k}{(2\pi)^3} \left\{ \frac{\partial E_e^+(\Delta)}{\partial |\mathbf{Q}|} \tanh \left[\frac{E_e^+(\Delta)}{2T} \right] + \frac{\partial E_e^-(\Delta)}{\partial |\mathbf{Q}|} \tanh \left[\frac{E_e^-(\Delta)}{2T} \right] \right\}, \quad (11)$$

with

$$\frac{\partial E_e^\pm(\Delta)}{\partial |\mathbf{Q}|} = \left[\frac{e}{2} \pm \frac{E_{S,e}(|\mathbf{k}|, |\mathbf{Q}|, \theta, \bar{\mu})}{E_e^+(\Delta) - E_e^-(\Delta)} \right] \frac{Q + 2k \cos \theta}{2|2\mathbf{k} + \mathbf{Q}|} - \left[\frac{e}{2} \mp \frac{E_{S,e}(|\mathbf{k}|, |\mathbf{Q}|, \theta, \bar{\mu})}{E_e^+(\Delta) - E_e^-(\Delta)} \right] \frac{Q - 2k \cos \theta}{2|2\mathbf{k} - \mathbf{Q}|}. \quad (12)$$

The zero temperature counterparts of Eqs. (10)-(11) are straightforward to obtain with the help of the identity $\tanh(x/2) = 1 - 2f(x)$ and the limiting form of the Fermi distribution function $f(x) = \theta(-x)$ for $T \rightarrow 0$, where $\theta(x)$ is the Heaviside step function.

As long as strange quarks are too heavy to appear in matter, the net positive charge of quarks is neutralized by electrons. At relevant densities and temperatures the electrons can be treated to a good approximation as a noninteracting gas. Their thermodynamic potential is then given by

$$\Omega_{\text{elec}} = 2T \int \frac{d^3 k}{(2\pi)^3} [\ln f(-\xi_{\text{elec}}^-(\mathbf{k}, \mu_e)) + \ln f(-\xi_{\text{elec}}^+(\mathbf{k}, \mu_e))], \quad (13)$$

with $\xi_{\text{elec}}^-(\mathbf{k}, \mu_e) = \sqrt{k^2 + m_e^2} - \mu_e$, where m_e is the electron mass. The second term in Eq. (13) represents the contribution of anti-particles (positrons) to the thermodynamical potential with $\xi_{\text{elec}}^+(\mathbf{k}, \mu_e) = \sqrt{k^2 + m_e^2} + \mu_e$.

III. IMPOSING CHARGE AND COLOR NEUTRALITY

The matrix representing the chemical potentials of quarks in the color-flavor space can be written in terms of the color group generators and the matrix of electrical charge operators as

$$\mu = \frac{\mu_B}{3} - \mu_e Q_e + \mu_3 T_3^c + \mu_8 T_8^c, \quad (14)$$

where μ_B is the baryon chemical potential, the electrical charge operator in flavor space is

$$Q_e = \text{diag} \left(\frac{2}{3}, -\frac{1}{3}, -\frac{1}{3} \right), \quad (15)$$

while the color operators are given by

$$T_3^c = \frac{1}{2} \text{diag}(1, -1, 0), \quad T_8^c = \frac{1}{2\sqrt{3}} \text{diag}(1, 1, -2). \quad (16)$$

We choose the direction of the color symmetry breaking the blue direction. The densities of red and green quarks are then automatically equal, therefore $n_3 \equiv 0$ and there is no need for the conjugate to n_3 chemical potential μ_3 . From Eqs. (14)-(16),

$$\bar{\mu} = \frac{\mu_B}{3} - \frac{\mu_e}{6} + \frac{\mu_8}{2\sqrt{3}}, \quad (17)$$

$$\delta\mu = -\frac{\mu_e}{2}. \quad (18)$$

The full thermodynamic potential of our system is now given by $\Omega = \Omega_{\text{MF}} + \Omega_{\text{elec}}$. By choosing instead of chemical potentials of quarks the chemical potentials μ_8 and μ_e as independent thermodynamic variables we obtain a set of equations that should be solved simultaneously:

$$\frac{\partial\Omega_{\text{MF}}}{\partial\Delta} = 0, \quad \frac{\partial\Omega_{\text{MF}}}{\partial|\mathbf{Q}|} = 0, \quad (19)$$

$$\frac{\partial\Omega}{\partial\mu_e} = 0, \quad \frac{\partial\Omega}{\partial\mu_8} = 0. \quad (20)$$

The densities of quarks and leptons can be derived through derivatives of the thermodynamic potential with respect to the conjugate chemical potentials; for electron density we have

$$n_{\text{elec}} = -\frac{\partial\Omega_{\text{elec}}}{\partial\mu_e} = 2 \int \frac{d^3k}{(2\pi)^3} [f(\xi_{\text{elec}}^+) - f(\xi_{\text{elec}}^-)]. \quad (21)$$

The densities of paired up and down quarks are given by

$$n_d^{rg} + n_u^{rg} = -\frac{\partial\Omega}{\partial\bar{\mu}} = -\sum_e \int \frac{d^3k}{(2\pi)^3} \frac{eE_{S,e}}{\sqrt{E_{S,e}^2 + \Delta^2}} \left[\tanh\left(\frac{E_e^+(\Delta)}{2T}\right) - \tanh\left(\frac{E_e^-(\Delta)}{2T}\right) \right], \quad (22)$$

$$n_u^{rg} - n_d^{rg} = -\frac{\partial\Omega}{\partial\delta\mu} = \sum_e \int \frac{d^3k}{(2\pi)^3} \left[\tanh\left(\frac{E_e^+(\Delta)}{2T}\right) + \tanh\left(\frac{E_e^-(\Delta)}{2T}\right) \right]. \quad (23)$$

The densities of unpaired up and down quarks are given by

$$n_d^b = 2 \sum_e \int \frac{d^3k}{(2\pi)^3} ef(\xi_e(\mathbf{k}, \mu_d^b)), \quad (24)$$

$$n_u^b = 2 \sum_e \int \frac{d^3k}{(2\pi)^3} ef(\xi_e(\mathbf{k}, \mu_u^b)). \quad (25)$$

The total densities of up and down quarks are

$$n_d = n_d^{rg} + n_d^b, \quad (26)$$

$$n_u = n_u^{rg} + n_u^b. \quad (27)$$

The baryon density is

$$n_B = \frac{n_u + n_d}{3}. \quad (28)$$

The electrical and charge neutrality require, respectively,

$$\frac{2}{3}n_u - \frac{1}{3}n_d - n_{\text{elec}} = 0, \quad (29)$$

$$n_d^{rg} + n_u^{rg} - 2n_d^b - 2n_u^b = 0. \quad (30)$$

The thermodynamic properties of matter are completely determined through four gap equations (19)-(20) for a given temperature and baryon density (or baryon chemical potential).

TABLE I: The column entries are as follows: baryonic density in units of fm^{-3} , total pressure and electron pressure in units of MeV fm^{-3} , the baryonic chemical potential, the chemical potentials of u , d quarks and electrons in units of MeV , the densities of d quarks, u quarks, and electrons in units of fm^{-3} , the gap in units of MeV , and total momentum of the condensate in units of fm^{-1} .

n_B	P_{tot}	P_e	μ_B	μ_d	μ_u	μ_e	n_d	n_u	n_e	Δ	$ \mathbf{Q} $
$T = 1 \text{ MeV}$											
0.32	63.9	27.9	1316.0	498.7	474.1	24.6	0.43	0.55	0.14	38.0	0.21
0.36	76.5	32.6	1370.6	519.6	494.0	25.6	0.49	0.61	0.16	45.8	0.29
0.40	90.2	37.6	1422.0	539.4	512.9	26.5	0.55	0.68	0.18	53.7	0.37
0.44	105.0	42.7	1470.8	558.2	530.9	27.4	0.62	0.75	0.20	61.9	0.45
0.48	121.0	47.9	1517.2	576.3	548.1	28.2	0.68	0.81	0.21	70.2	0.53
$T = 5 \text{ MeV}$											
0.32	66.5	29.0	1322.5	501.5	475.8	25.7	0.44	0.55	0.14	37.8	0.00
0.36	77.1	32.8	1369.4	519.2	493.3	25.9	0.49	0.61	0.16	45.7	0.00
0.40	90.7	37.6	1420.1	538.6	512.0	26.6	0.55	0.67	0.18	53.7	0.00
0.44	105.6	42.7	1468.4	557.2	529.7	27.5	0.61	0.74	0.20	61.8	0.18
0.48	121.7	48.0	1514.7	575.1	546.8	28.3	0.67	0.80	0.21	70.2	0.29
$T = 10 \text{ MeV}$											
0.32	62.7	26.7	1309.5	495.6	474.6	21.0	0.43	0.53	0.14	35.9	0.00
0.36	86.6	39.3	1370.8	523.1	488.0	35.2	0.46	0.62	0.16	43.6	0.00
0.40	100.4	44.1	1422.8	542.5	507.7	34.8	0.53	0.69	0.18	52.7	0.00
0.44	111.3	46.3	1468.9	558.8	527.6	31.1	0.60	0.74	0.19	61.4	0.00
0.48	126.1	50.5	1513.9	575.6	545.2	30.4	0.66	0.80	0.21	70.0	0.00

IV. RESULTS

Eqs. (19) and (20) were solved numerically by finding a solution that satisfies simultaneously the four integral equations. The ultraviolet divergence of the integrals was regularized by a three-dimensional cut-off in momentum space $|\mathbf{p}| < \Lambda$. The phenomenological value of the coupling constant $G = g^2/12\Lambda^2$ in the $\langle qq \rangle$ Cooper channel is related to the coupling constant G_d in the $\langle \bar{q}q \rangle$ di-quark channel by the relation $G = N_c/(2N_c - 2)G_d$, where $N_c = 3$ is the number of quark colors. The coupling constant G_d and the cut-off Λ are fixed by adjusting the model to the vacuum mass and decay constant of the pion. As in Ref. [20] we employ the parameter values $G_d = 3.11 \text{ GeV}^{-2}$ and $G_d\Lambda^2 = 1.31$.

The phase diagram of charge and color neutral matter is shown in Fig. 1. The baryonic density range extends up to $3\rho_0$, where $\rho_0 = 0.16 \text{ fm}^{-3}$ is the nuclear saturation density. The model is limited to the above density range, because at somewhat larger densities the Fermi momentum becomes comparable to the cut-off of the model. The covered temperature range extends up to the critical temperature for the onset of color superconducting phase transition. It is seen that the low-temperature domain is occupied by the FF phase; in the intermediate temperature regime $5 \leq T \leq 20 \text{ MeV}$, the FF phase is replaced by a homogeneous 2SC phase, the phase transition being first order. At the critical temperature, which depending on the density lies in the range $T_c \sim 20 - 30 \text{ MeV}$, there is a second order phase transition from the 2SC to the unpaired phase.

The equation of state of matter and related parameters are tabulated in Table I. The large pairing gaps at large densities are due to the density scaling of the density of states at the Fermi surface; note that the magnitude of the gap depends on the chosen value of the cutoff and is, therefore, specific to the present model. Furthermore, the pressure requires some normalization procedure, which is chosen to be the standard NJL prescription, which requires zero pressure (and energy density) in vacuum (*i.e.*, at $T = \mu = 0$). In practice, the low density limit of the model should correspond to quarks confined in baryons, so that the normalization should be controlled by the confinement mechanism in non-zero temperature/density QCD. This freedom allows one to add a bag constant to the pressure of the NJL model to match it to the equation of state of hadronic matter at a desired density and temperature

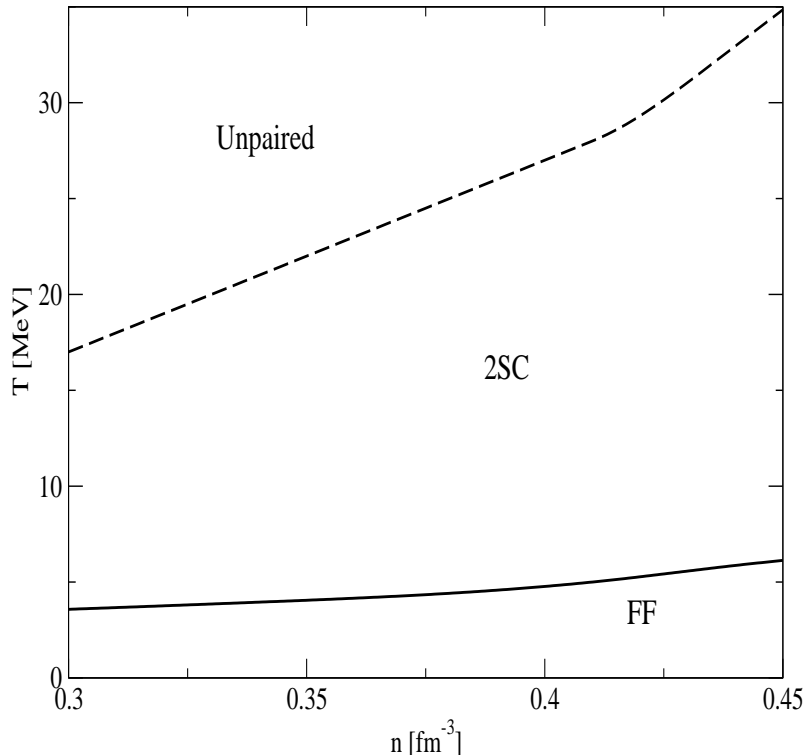


FIG. 1: (color online). The phase diagram of neutral two-flavor quark matter. The high-temperature low-density region corresponds to unpaired quark matter, the intermediate temperature region corresponds to the homogeneous 2SC phase, and the low-temperature domain corresponds to the inhomogeneous FF phase of quark matter.

of deconfinement. The point of onset of deconfinement is, thus, a free parameter of the model. Consequently, the pressure of the model may be shifted by a constant value of the bag.

As seen from Table I, the equation of state is only weakly temperature dependent. Table I shows also the composition of matter and the chemical potentials of species, which are related by the β -equilibrium condition $\mu_d = \mu_e + \mu_u$. This condition will be violated in the high temperature domain $T \geq 10$ MeV of our phase diagram, where neutrinos are thermally populated as a consequence of their short mean-free path at such temperatures. The non-zero neutrino chemical potential then must be added to the left-hand side of the β -equilibrium condition (18). Since there is no or little overlap between the domain occupied by the FF phase and the domain where neutrinos are trapped, we shall not repeat our analysis at non-zero neutrino chemical potential. Let us note that the phase transition from the 2SC to the FF phase is first order and is associated with a latent heat; such a heat release in compact stars during the collapse stage may serve as an engine that powers high-energy burst phenomena from the collapsing star.

V. SUMMARY

In this paper we have investigated the phase diagram of charge and color neutral quark matter in two-flavor NJL model. Our results strongly suggest that the 2SC phase does not exist in the interiors of mature compact stars. Instead, the stellar matter at intermediate densities is in the FF, or related crystalline, phase. A possible alternative, which is not excluded by our calculations, is that nuclear matter directly transforms to three-flavor CFL matter. The FF phase provides an upper bound on the energy of the color superconducting phase, which can presumably be lowered by choosing more complicated forms of the order parameter. The model parameters in our study were chosen to correspond to the common values adopted in the NJL model; in the unlikely case where the in-medium couplings are substantially larger, the 2SC phase may have lower energy than the FF phase (see Ref. [20]).

Provided a nuclear equation of state and a matching procedure (*e.g.*, Maxwell construction) the present equation

of state can be used to construct the complete equation of state of matter in compact stars and their models (such constructions are reviewed in, *e.g.*, Refs. [26, 27]). The interesting possibility that three-flavor crystalline phase may appear in stable massive compact stars has been pointed out recently [25]. The presence of a crystallinelike phase in the interiors of hybrid compact stars may have a number of interesting astrophysical implications; *e.g.*, the finite shear modulus of matter implies deformations of the quark core that can generate gravitational waves at the level of the sensitivity of current gravitational wave observatories [28]. The properties of the FF phase may manifest themselves in the cooling of compact stars. The emission rate of the dominant Urca process has been computed in phases similar to the FF phase in Refs. [29, 30]. Finally, the first order BCS-FF phase transition and the latent heat release associated with it may power high-energy burst phenomena from collapsing stars.

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